Suite 2, 179 Given Tce, Paddington, QLD, 4064
phone: 1300 360 245 | fax: 1300 360 237
email: contact@personaltutors.com.au
web: www.personaltutors.com.au

UNIT 3

Topic 1: The logarithmic function 2

1 Logarithmic laws and logarithmic functions

1.1 Establish and use logarithmic laws and definitions

Logarithms can only be used when the base and exponent are positive numbers ($a > 0$ and $N > 0$). For any other values, the logarithm is not defined.

If $N = a^x$, then $x = log_a N$.

A natural logarithm uses Euler's number (e) as its base number.

If $N = e^x$, then $x = log_e N$, also written as $x = ln N$.

The logarithmic laws are:

- $log_a(m) + log_a(n) = log_a(mn)$
- $log_a(m) - log_a(n) = log_a\left(\frac{m}{n}\right)$
- $log_a(m)^n = n\, log_a(m)$
- $log_a(1) = 0$
- $log_a(a) = 1$
- $log_a(0)$ is undefined
- $a^{log_a(m)} = m$
- Change of base rule: $log_a(m) = \frac{log_b(m)}{log_b(a)}$

Suite 2, 179 Given Tce, Paddington, QLD, 4064
phone: 1300 360 245 | fax: 1300 360 237
email: contact@personaltutors.com.au
web: www.personaltutors.com.au

1.2 Interpret and use logarithmic scales such as decibels in acoustics, the Richter scale for earthquake magnitude, octaves in music, pH in chemistry

Types of logarithmic scales:

- Decibels in acoustics
- The Richter scale for earthquake magnitude
- Octaves in music
- pH in chemistry
- Any real-life situation in which there is continuous growth or decay over time

1.3 Solve equations involving indices with and without technology

Review of Index Laws

Index form: a^x, with base, a, and the index, x (also called the logarithm, power or exponent).

The index laws are:

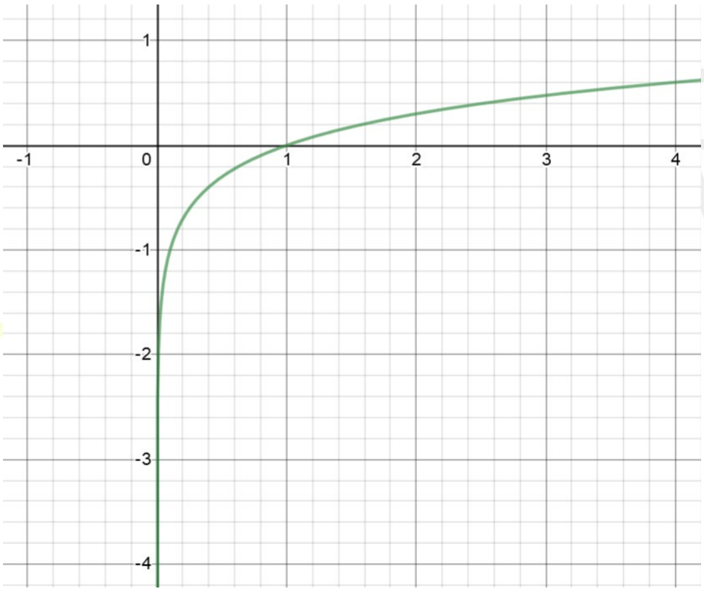
- When numbers with the same base are multiplied, the indices are added.
  $a^x \times a^y = a^{x+y}$
- When numbers with the same base are divided, the indices are subtracted.
  $a^x \div a^y = a^{x-y}$
- When numbers with an index or exponent are raised to another index or exponent, the indices are multiplied.
  $(a^x)^y = a^{xy}$
- When numbers have an index of 0, the answer is 1.
  $a^0 = 1$
- When a number has a negative index, it becomes a fraction with a positive index.
  $a^{-x} = \frac{1}{a^x}$ and $\frac{1}{a^{-x}} = a^x$
- When a number has a fractional index, the denominator of the fraction becomes the root.
  $a^{\frac{1}{y}} = \sqrt[y]{a}$ and $a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Suite 2, 179 Given Tce, Paddington, QLD, 4064
phone: 1300 360 245 | fax: 1300 360 237
email: contact@personaltutors.com.au
web: www.personaltutors.com.au

1.4 Recognise the qualitative features of the graph of $y=\log_a(x)$ ($a>1$), including asymptotes, and of its translations $y=\log_a(x)+b$ and $y=\log_a(x+c)$

The graph of the logarithmic function $f(x) = log_a(x)$, $a > 1$ has the following characteristics:

- The domain is $(0, \infty)$.
- The range is R.
- The graph is an increasing function.
- The graph cuts the x -axis at $(1,0)$.
- As $x \to 0$, $y \to -\infty$, so the line $x = 0$ is an asymptote.
- As a increases, the graph rises more steeply for $x \in (0,1)$ and is flatter for $x \in (1, \infty)$.



Transformations of Logarithmic Graphs

1. Dilation

$y = n\,log_a(x)$ is dilated by factor n parallel to the y-axis or from the x -axis.
$y = log_a(mx)$ is dilated by factor $\frac{1}{m}$ parallel to the x-axis or from the y-axis.

2. Reflection

$y = -\,log_a(x)$ is reflected in the x -axis.
$y = log_a(-x)$ is reflected in the y -axis.

3. Vertical translation

$y = log_a(x) + k$ is translated k units parallel to the y -axis.

4. Horizontal translation

$y = log_a(x - h)$ is translated h units parallel to the x -axis.

1.5 Identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.

As logarithmic functions are essentially the inverse of exponential functions, they can be used to solve exponential functions of the form:

$$A = A_0 e^{kt}$$

Where:

- A_0 represents the initial value
- t represents the time taken
- k represents the rate constant

1.6 Solve equations involving logarithmic functions with and without technology

For example, to solve for t:

$$\frac{A}{A_0} = e^{kt}$$

$$log_e\left(\frac{A}{A_0}\right) = kt$$

$$t = \frac{1}{k} log_e\left(\frac{A}{A_0}\right)$$