

UNIT 3

Topic 1: Gravity and Motion

1 Vectors

1.1 Use vector analysis to resolve a vector into two perpendicular components

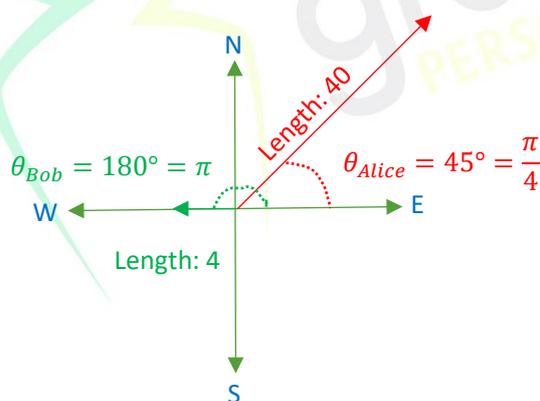
Vector: A quantity that has both magnitude and direction; a vector may be represented pictorially by an arrowed line segment (\rightarrow) and symbolically as \vec{v} .

E.g. vectors are used to store quantities which are described with a number AND direction (e.g. 60km/hr travelling North-East) not just a number (e.g. 60km/hr).

Vectors are visually represented:

- On an axis, as arrows.
- The magnitude is shown by the length of the arrow.
- The direction is shown by the way the arrow is facing.
 - The direction is usually described by an angle θ from the positive horizontal axis.

E.g. If Alice is driving at 40km/hr North-East (shown in red), and Bob is walking at 4km/hr due West (shown in green), then the corresponding vectors can be shown as:



Vectors can be split into two, independent, perpendicular components (most frequently horizontal and vertical) using trigonometry.

If the magnitude of the vector is given by r , and its direction is given by the angle from the positive x -axis, θ , then its resolution into horizontal and vertical components is given by the following equations:

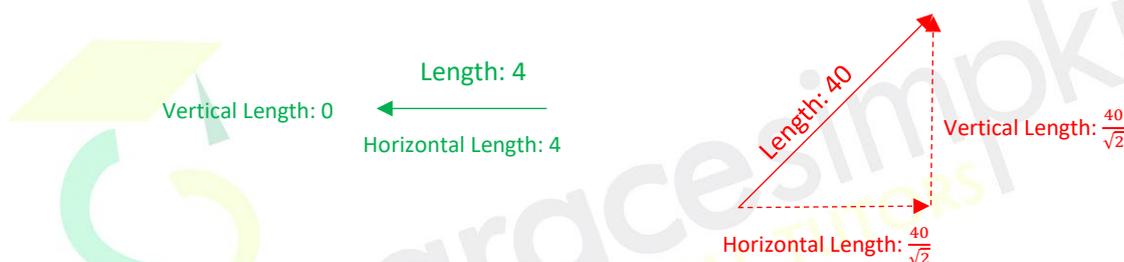
$$\vec{v} = r\cos(\theta)\vec{i} + r\sin(\theta)\vec{j}$$

E.g. from before, Bob is shown in green, and Alice is shown in red.

$$\vec{v}_{Bob} = 4\cos(\pi)\vec{i} + 4\sin(\pi)\vec{j}, \quad \vec{v}_{Alice} = 40\cos\left(\frac{\pi}{4}\right)\vec{i} + 40\sin\left(\frac{\pi}{4}\right)\vec{j}$$

$$\vec{v}_{Bob} = -4\vec{i} + 0\vec{j}, \quad \vec{v}_{Alice} = \frac{40}{\sqrt{2}}\vec{i} + \frac{40}{\sqrt{2}}\vec{j}$$

This split can be visualised as:



1.2 Solve vector problems by resolving vectors into components, adding or subtracting the components and recombining them to determine the resultant vector.

Once resolved into components, vectors can be added or subtracted.

Addition of Vectors:

- To perform addition, add the \vec{i} components of each of the two vectors together, and then make that result the \vec{i} component of their sum.
- Repeat for \vec{j} components.

Example

Let vector 1 be: $\vec{v}_1 = a\vec{i} + b\vec{j}$

Let vector 2 be: $\vec{v}_2 = c\vec{i} + d\vec{j}$

$$\vec{v}_1 + \vec{v}_2 = (a + c)\vec{i} + (b + d)\vec{j}$$

Subtraction of Vectors:

- To perform subtraction, do the same process as addition, however instead of **adding** the two components, you should **subtract** them instead.

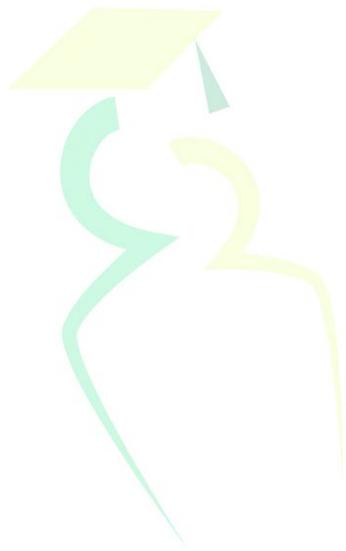
Example

Let vector 1 be: $\vec{v}_1 = a\vec{i} + b\vec{j}$

Let vector 2 be: $\vec{v}_2 = c\vec{i} + d\vec{j}$

$$\vec{v}_1 - \vec{v}_2 = (a - c)\vec{i} + (b - d)\vec{j}$$

$$\vec{v}_2 - \vec{v}_1 = (c - a)\vec{i} + (d - b)\vec{j}$$



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